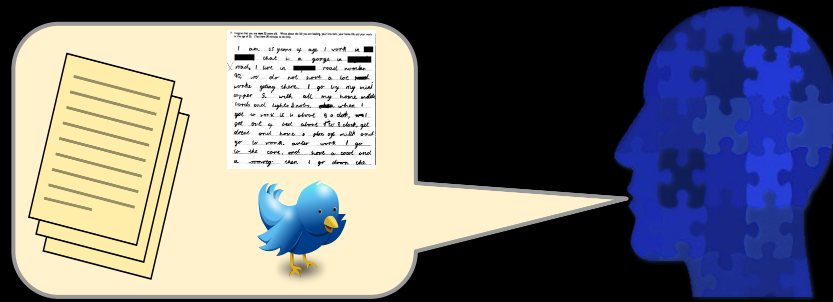
Supervised Classification: Logistic Regression

CSE354 - Spring 2020
Special Topic in CS

NLP's practical applications

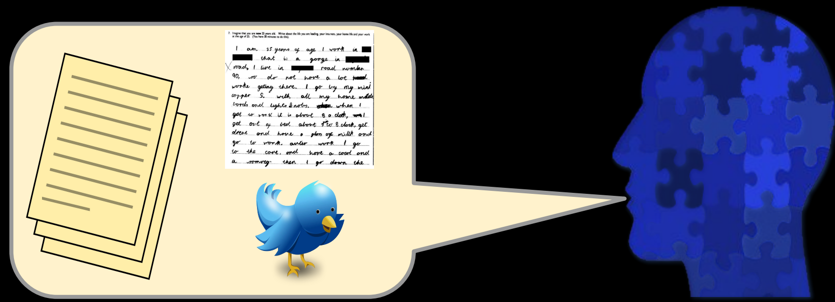


- Machine translation
- Automatic speech recognition
  - Personalized assistants
  - Auto customer service
- Information Retrieval
  - Web Search
  - Question Answering
- Sentiment Analysis
- Computational Social Science
- *Growing day by day*

how?

- Machine learning:
  - Logistic regression
  - Probabilistic modeling
  - Recurrent Neural Networks
  - Transformers
- Algorithms, e.g.:
  - Graph analytics
  - Dynamic programming
- Data science
  - Hypothesis testing

NLP's practical applications



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Topics we will cover

- Supervised Classification
- Goal of logistic regression
- The “loss function” -- what logistic regression tries to optimize
- Adding Multiple Features
- Training and Test Sets
- Overfitting; Role of Regularization

Supervised Classification

X - features of N observations (i.e. words)

Y - class of each of N observations

GOAL: Produce a *model* that outputs the most likely class y_i, given features x_i.

$$f(X) = Y$$

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Supervised Classifica

Some function or rules to go from *X* to *Y*, as close as possible.

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Supervised Classification

Supervised Machine Learning: Build a model with examples of outcomes (i.e. Y) that one is trying to predict.

Classification: The outcome (Y) is a discrete class (e.g. {noun, verb, adjective, adverb}; {positive sentiment, negative sentiment}).

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Binary classification goal: Build a model that can estimate P(A=1|B=?)

i.e. given B, yield (or “predict”) the probability that A=1

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Example: Y: 1 if target is verb, 0 otherwise;
X: 1 if “was” occurs before target; 0 otherwise

I was reading for NLP. *We were fine.* *I am good.*

The cat was very happy. *We enjoyed the reading material.* *I was good.*

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Example: Y: 1 if target is a part of a proper noun, 0 otherwise;
X: number of capital letters in target and surrounding words.

They attend Stony Brook University. *Next to the brook Gandalf lay thinking.*

The trail was very stony. *Her degree is from SUNY Stony Brook.*

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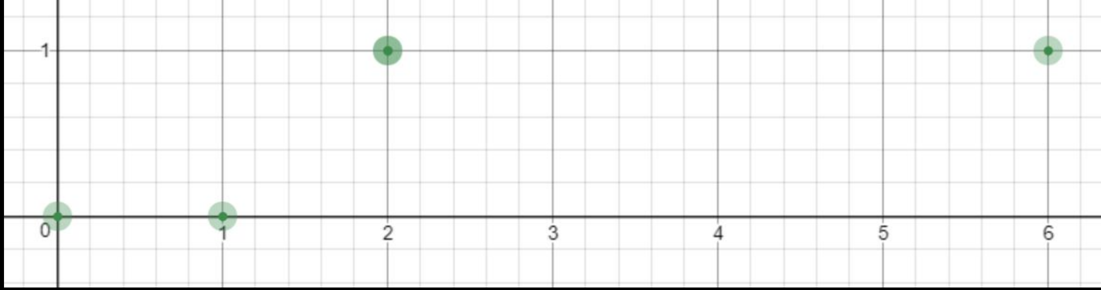
x	y
2	1
1	0
0	0
6	1
2	1

Logistic Regression



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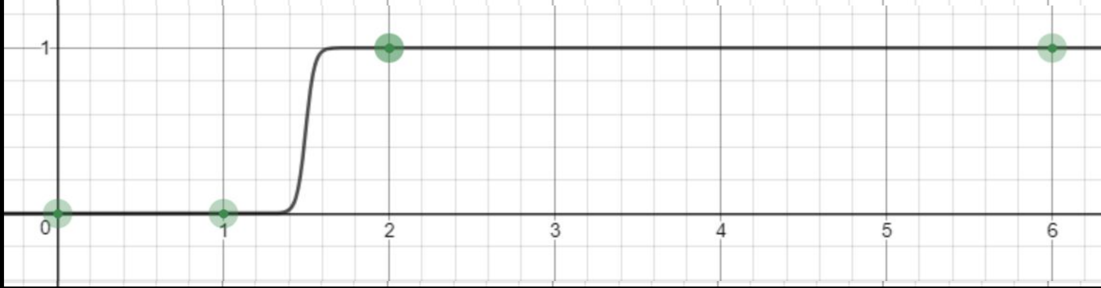

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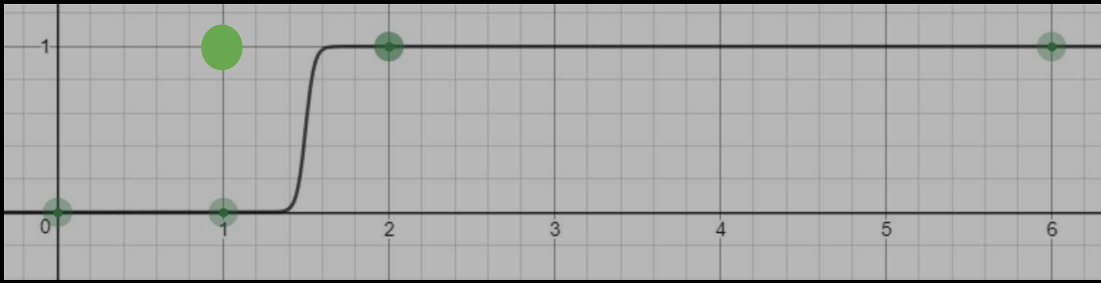

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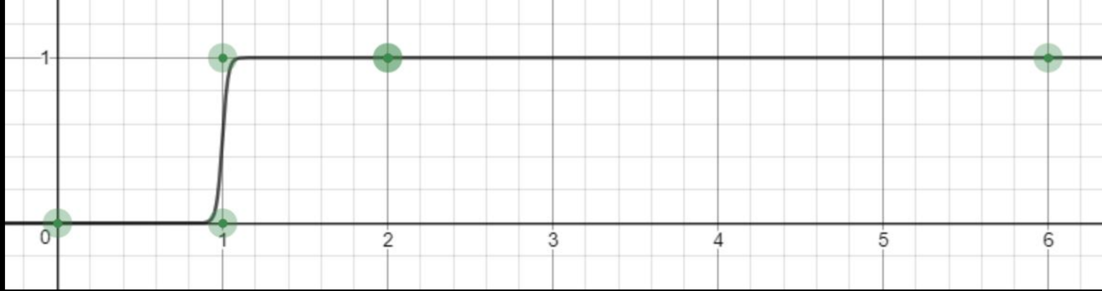

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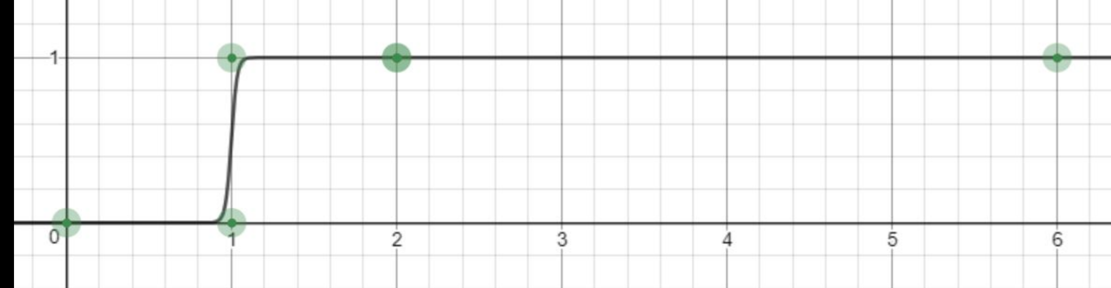

Logistic Regression

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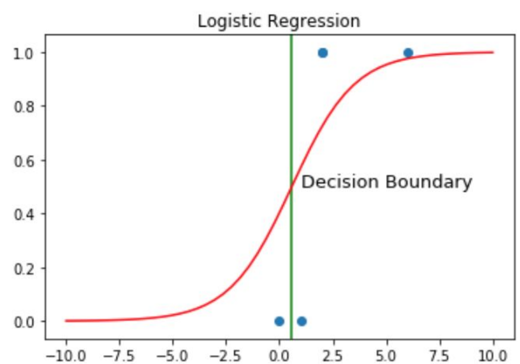
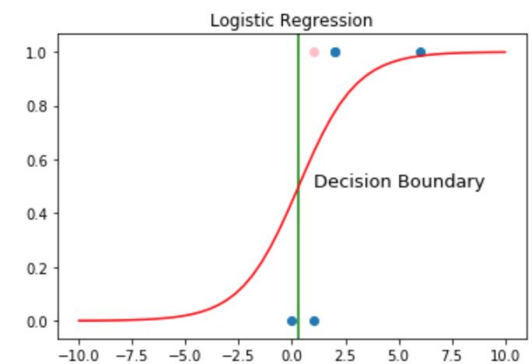


In [78]:
```
-b_0/b_1
```
Out[78]: 0.5824799517820446

In [28]:
```
logisticRegr.predict(x)
```
Out[28]: array([1, 1, 0, 1, 1])

Out[80]: [<matplotlib.lines.Line2D at 0x11a60f160>]



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In [82]:
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logisticRegr2.predict(x2)
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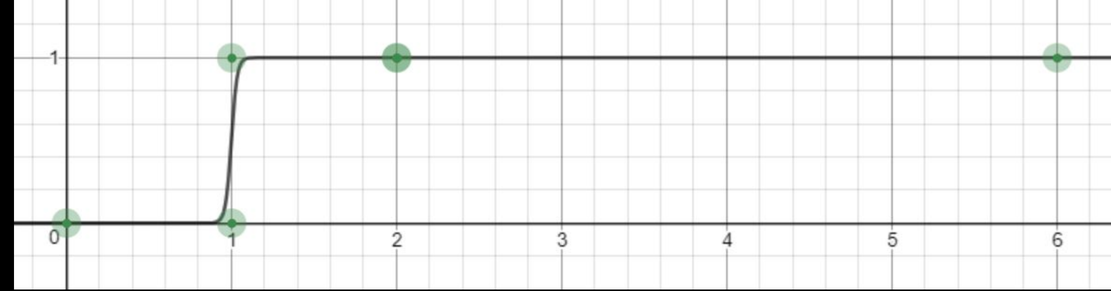
1	1

Logistic Regression



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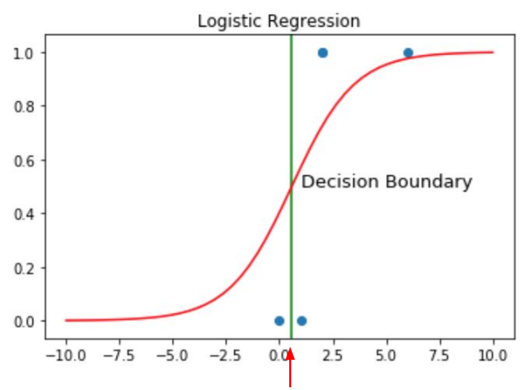


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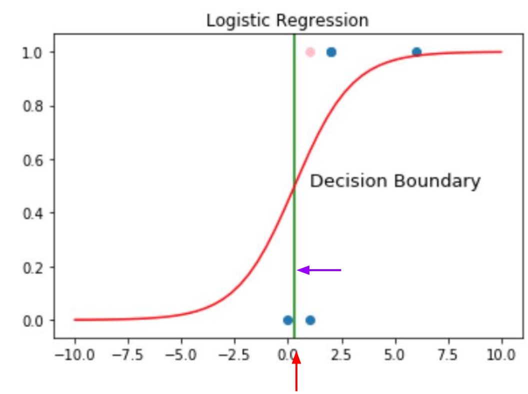


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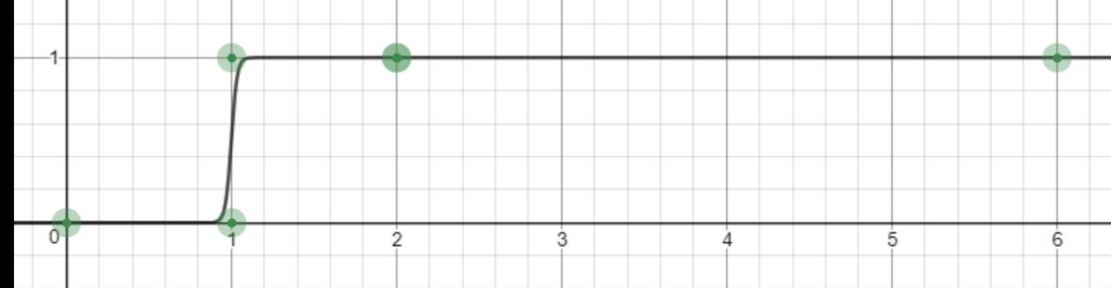

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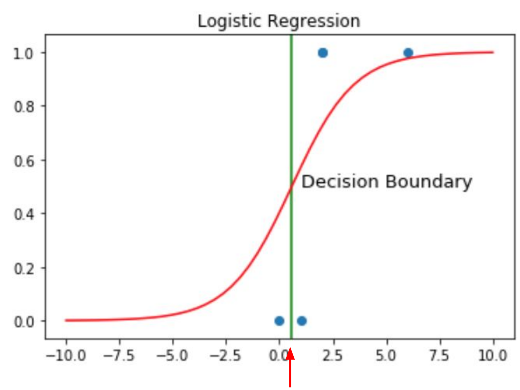


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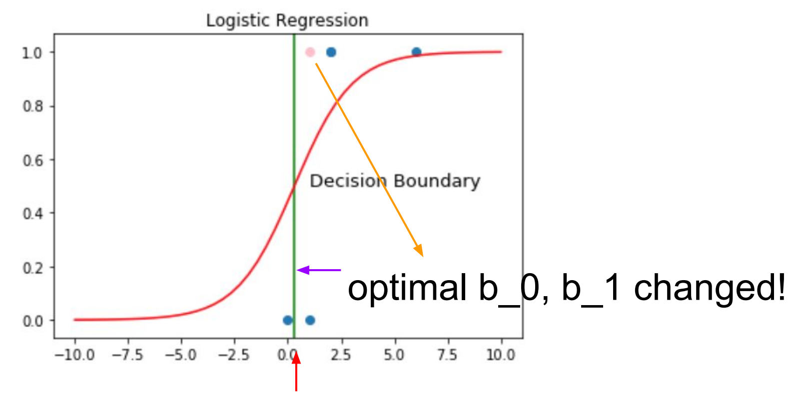


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Logistic Regression on a single feature (*x*)

$Y_i \in \{0, 1\}$; X is a **single value** and can be anything numeric.

$$p_i \equiv P(Y_i = 1 | X_i = x) = \frac{e^{\beta_0 + \beta_1 x_i}}{1 + e^{\beta_0 + \beta_1 x_i}}$$

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HOW? Essentially, try different B_0 and B_1 values until "best fit" to the training data (example X and Y).

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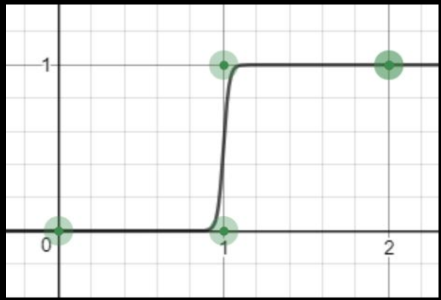

"best fit" : whatever maximizes the likelihood function:

$$L(\beta_0, \beta_1 | X, Y) = \prod_{i=1}^{n} p(x_i)^{y_i}(1 - p(x_i))^{1-y_i}$$

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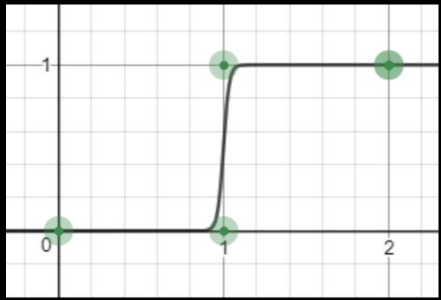

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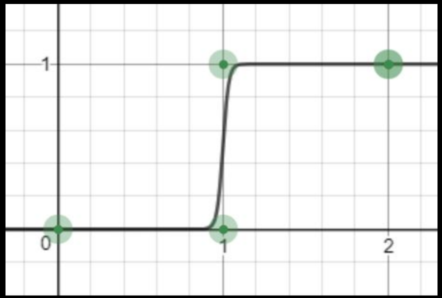
X can be multiple features

Often we want to make a classification based on multiple features:

- Number of capital letters surrounding: integer
- Begins with capital letter: {0, 1}
- Preceded by “the”? {0, 1}



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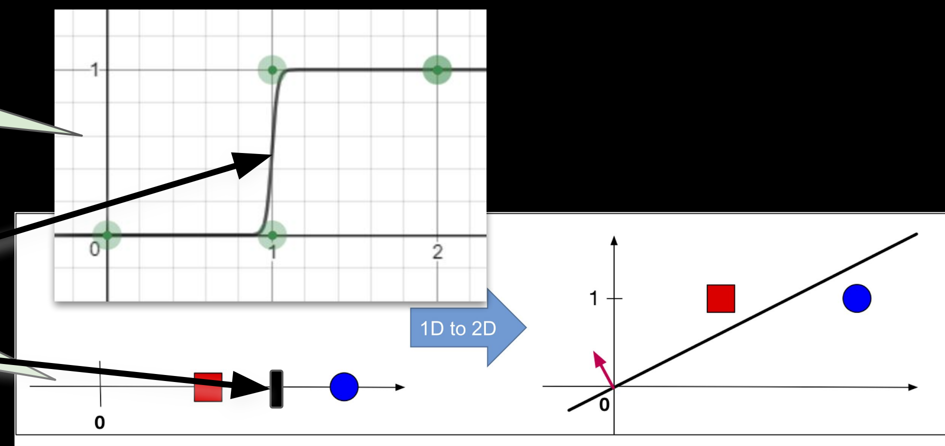
Y-axis is Y (i.e. 1 or 0)

To make room for multiple Xs, let’s get rid of y-axis. Instead, show **decision point**.



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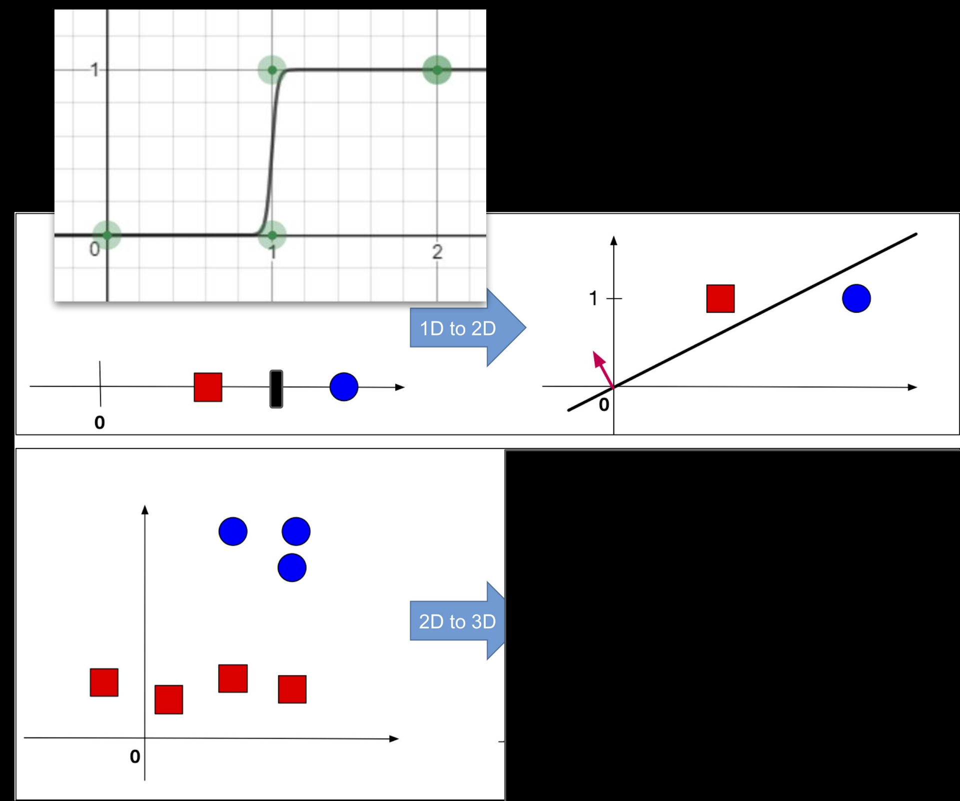

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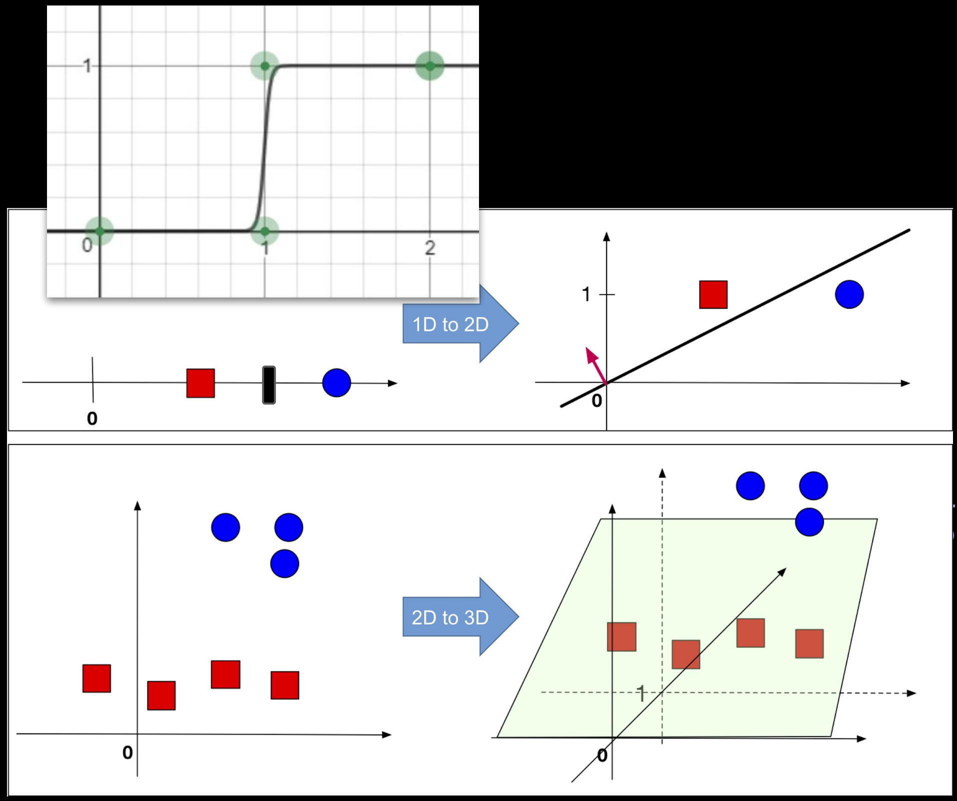

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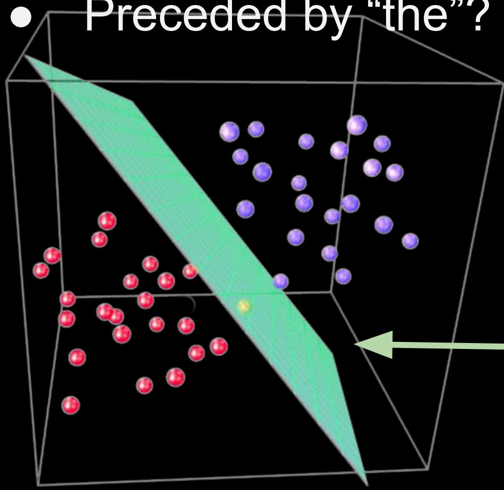

We’re learning a linear (i.e. flat) *separating hyperplane*, but fitting it to a *logit* outcome.

(https://www.linkedin.com/pulse/predicting-outcomes-probabilities-logistic-regression-konstantinidis/)

Logistic Regression

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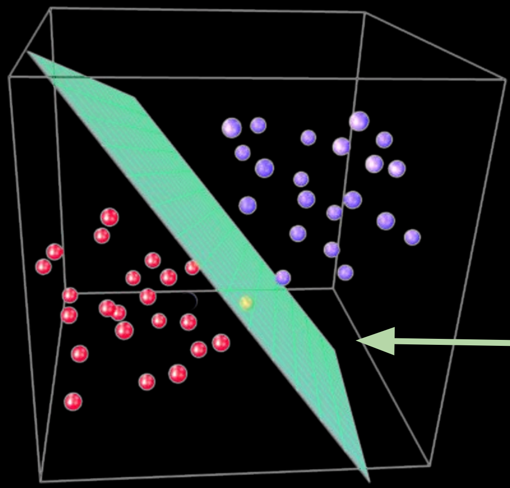

$$logit(p_i) = log\left(\frac{p_i}{1 - p_i}\right) = \beta_0 + \sum_{j=1}^{m} \boxed{\beta_j x_{ij}} = 0$$

We're still learning a linear *separating hyperplane*, but fitting it to a *logit* outcome.

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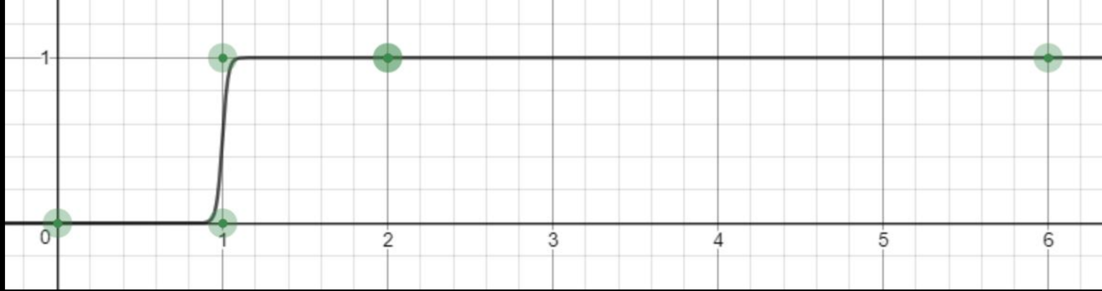

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They attend Binghamton.

x	y
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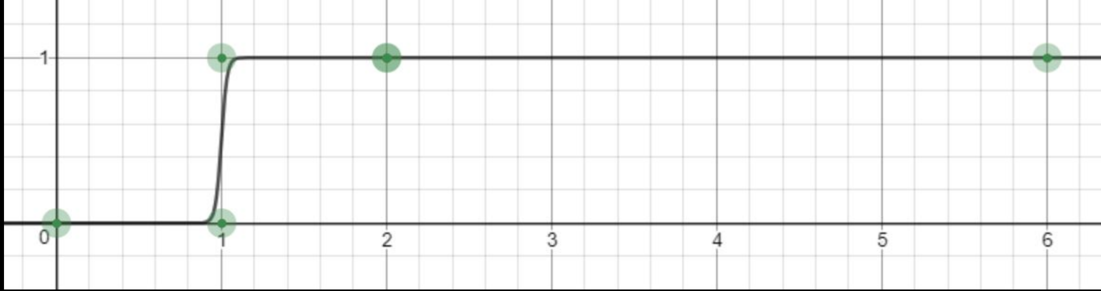
1	0
0	0
6	1
2	1
1	1

Logistic Regression



Example: Y: 1 if target is a part of a proper noun, 0 otherwise;

X1: number of capital letters in target and surrounding words.

Let's add a feature! X2: does the target word start with a capital letter?

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x2	x1	y
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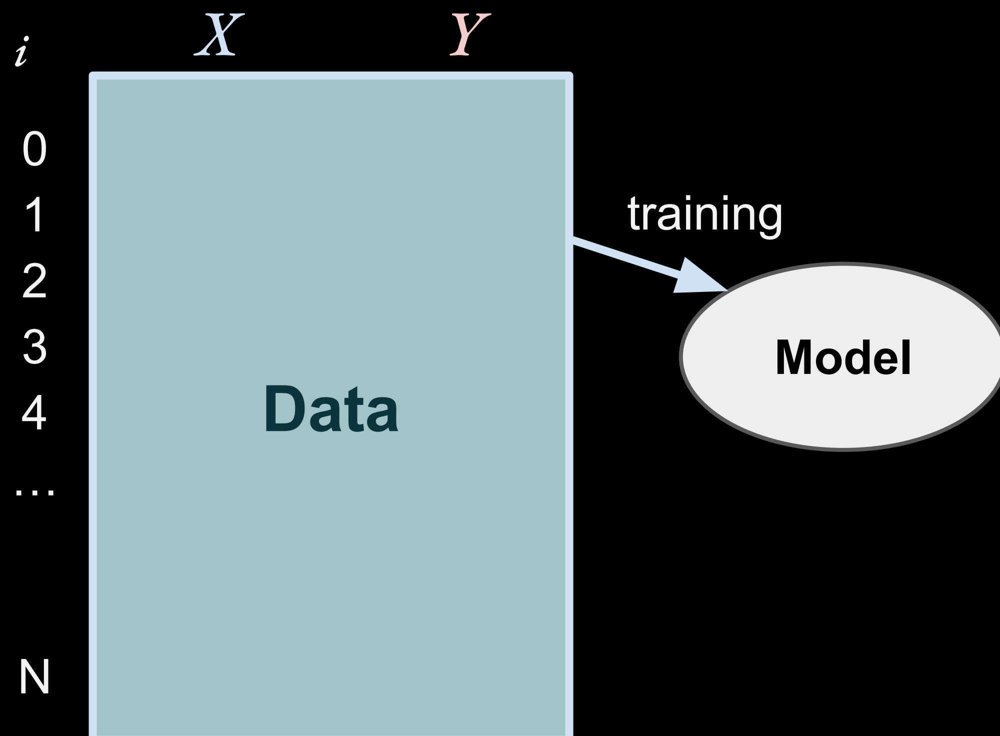
0	1	0
0	0	0
1	6	1
1	2	1
1	1	1

Machine Learning: How to setup data



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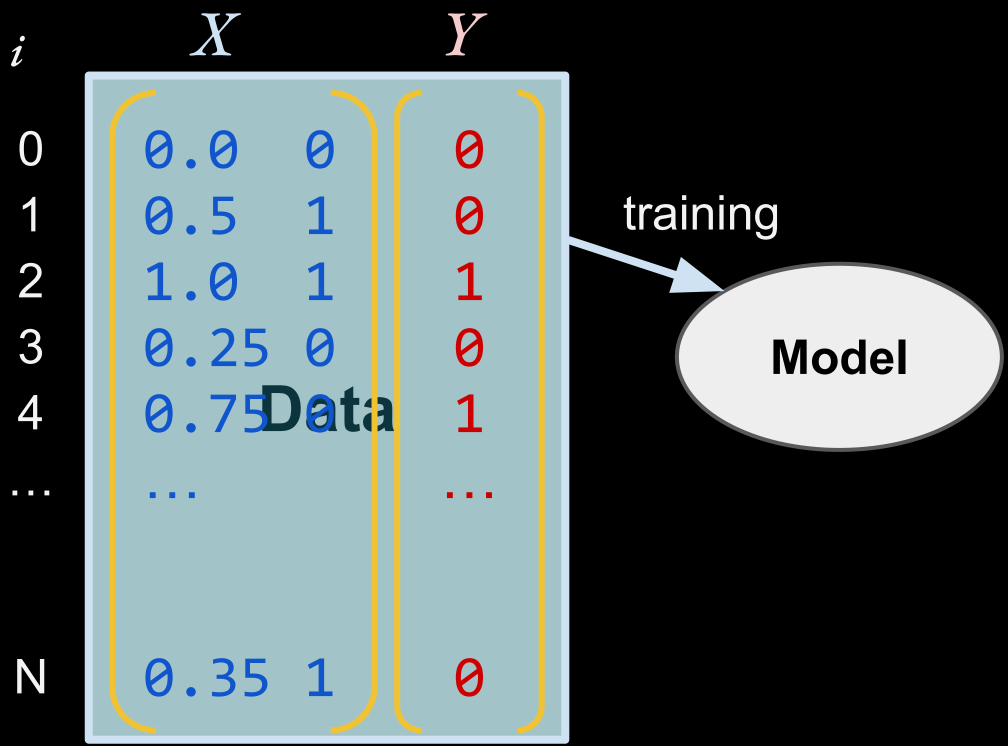

Machine Learning: How to setup data

“Corpus”

raw data:
sequences of characters

i	X		Y
0	0.0	0	0
1	0.5	1	0
2	1.0	1	1
3	0.25	0	0
4	0.75	0	1
…	…		…
N	0.35	1	0

Data

training

Machine Learning: How to setup data

Feature Extraction

--pull out *observations* and *feature vector* per observation.

"Corpus"

raw data: sequences of characters

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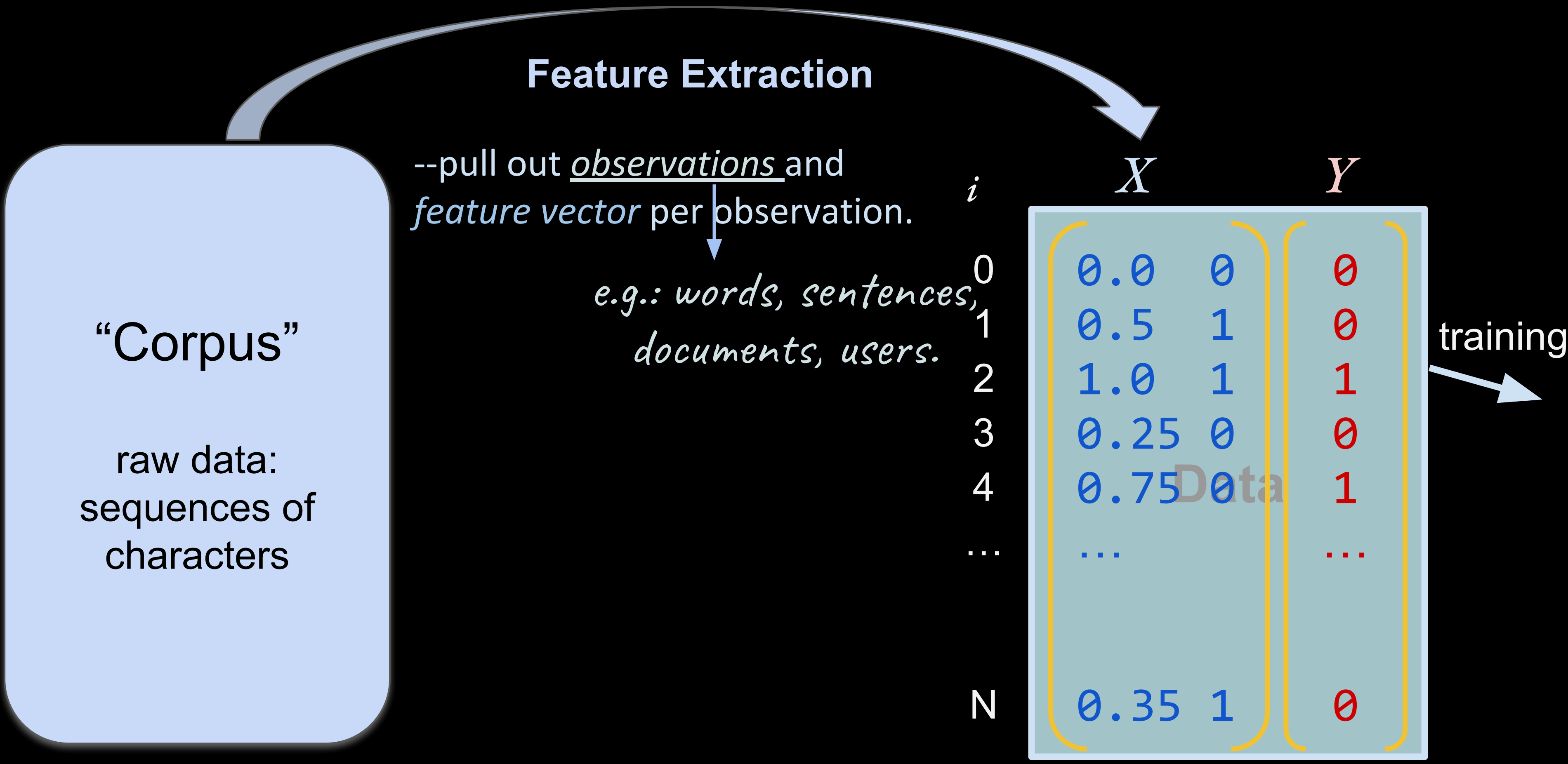
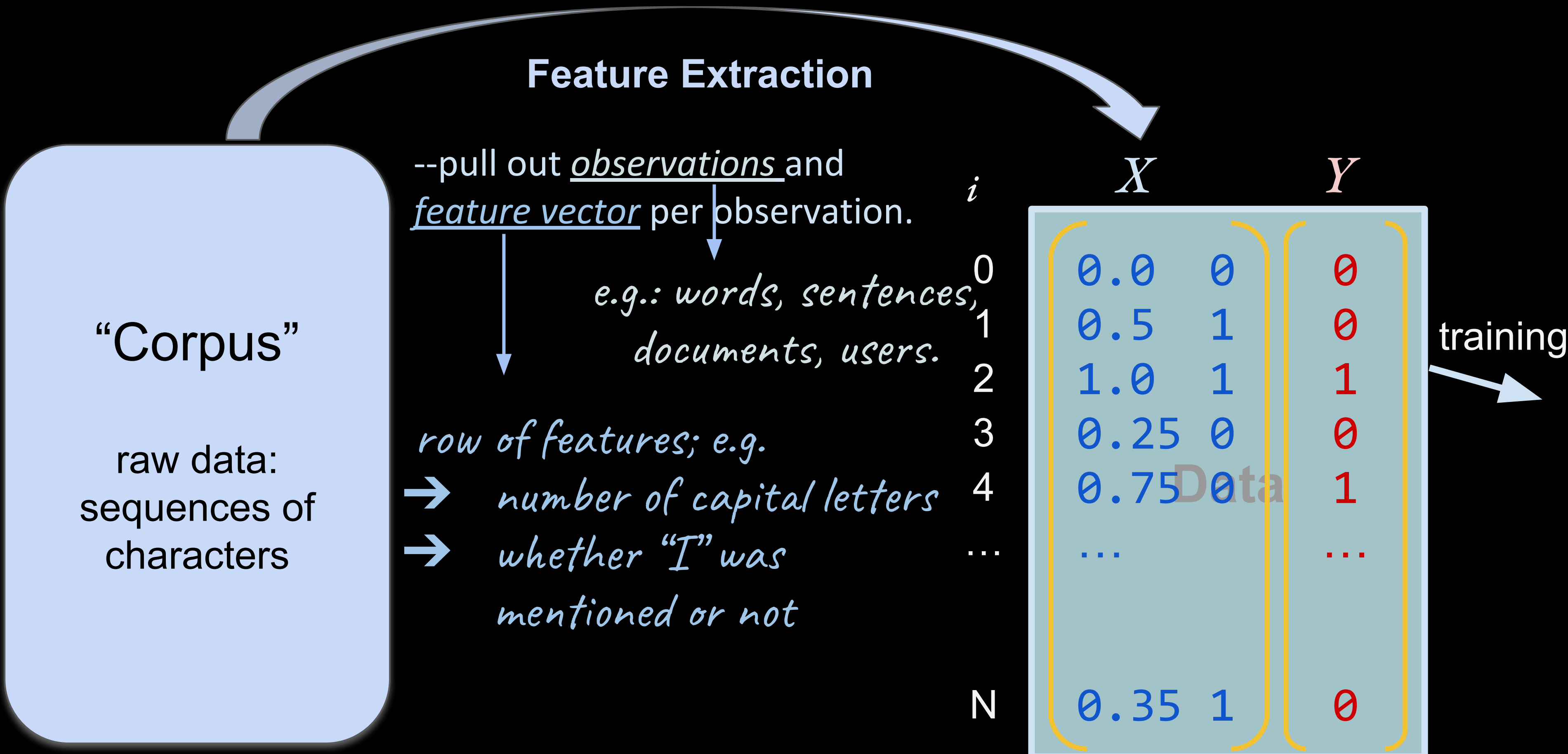


Machine Learning: How to setup data



Machine Learning: How to setup data

Feature Extraction

“Corpus”

raw data: sequences of characters

--pull out *observations* and *feature vector* per observation.

e.g.: words, sentences, documents, users.

row of features; e.g.

→ *number of capital letters*

→ *whether “I” was mentioned or not*

→ *k features indicating whether k words were mentioned or not*

Data

i	X		Y
0	0.0	0	0
1	0.5	1	0
2	1.0	1	1
3	0.25	0	0
4	0.75	0	1
…	…		…
N	0.35	1	0

training

Machine Learning: How to setup data

Feature Extraction

Multi-hot Encoding

- Each word gets an index in the vector
- 1 if present; 0 if not

"Corpus"

raw data: sequences of characters

of features; e.g.

→ number of capital letters

→ whether "I" was mentioned or not

→ *k features indicating whether k words were mentioned or not*

X Y

Data

Machine Learning: How to setup data

Feature Extraction

X Y

Multi-hot Encoding

- Each word gets an index in the vector
- 1 if present; 0 if not

Feature example: is word present in document?

The book was interesting so I was happy .

“Corpus”

raw data: sequences of characters

of features; e.g.

→ number of capital letters

→ whether “I” was mentioned or not

→ k features indicating whether k words were mentioned or not

Data

Machine Learning: How to setup data

Feature Extraction

X Y

“Corpus”

raw data: sequences of characters

Data

<u>Multi-hot Encoding</u>

- Each word gets an index in the vector
- 1 if present; 0 if not

Feature example: is word present in document?

The book was interesting so I was happy .

`[0, 1, 1, 0, 1, …, 1, 0, 1, 1, 0, 1, …, 1]`^{k}

→ *k features indicating whether k words were mentioned or not*

Machine Learning: How to setup data

Feature Extraction

X Y

Multi-hot Encoding

- Each word gets an index in the vector
- 1 if present; 0 if not

Feature example: is word present in document

"Corpus"

raw data: sequences of characters

The book was interesting so I was happy .

Data

[0, 1, 1, 0, 1, ..., 1, 0, 1, 1, 0, 1, ..., 1]^k

a

→ *k features indicating whether k words were mentioned or not*

sad

Machine Learning: How to setup data

Feature Extraction

X Y

Multi-hot Encoding

- Each word gets an index in the vector
- 1 if present; 0 if not

Feature example: is previous word "the"?

The ***book*** *was interesting so I was happy .*

[0, 1, 1, 0, 1, …, 1, 0, 1, 1, 0, 1, …, 1]^k

→ *k features indicating whether k words were mentioned or not*

"Corpus"

raw data: sequences of characters

Data

Machine Learning: How to setup data

Feature Extraction

"Corpus"

raw data: sequences of characters

X Y

Data

Multi-hot Encoding

- Each word gets an index in the vector
- 1 if present; 0 if not

Feature example: is previous word "the"?

*The **book** was interesting so I was happy .*

~~[0, 1, 1, 0, 1, …, 1, 0, 1, 1, 0, 1, …, 1]^k~~

→ k features indicating whether k words were mentioned or not

Machine Learning: How to setup data

Feature Extraction

X Y

One-hot Encoding

- Each word gets an index in the vector
- All indices 0 except present word:

  Feature example: is previous word “the”?

“Corpus”

raw data: sequences of characters

The ***book*** *was interesting so I was happy .*

`[0, 1, 0, 0, 0, …, 0, 0, 0, 0, 0, 0, …, 0]`^{k}

→ *k features indicating whether k words were mentioned or not*

Data

Machine Learning: How to setup data

Feature Extraction

X Y

“Corpus”

raw data: sequences of characters

Data

One-hot Encoding

- Each word gets an index in the vector
- All indices 0 except present word:

Feature example: which is previous word?

The book was interesting so I was happy .

[0, 1, 0, 0, 0, …, 0, 0, 0, 0, 0, 0, …, 0]^k

[0, 0, 1, 0, 0, …, 0, 0, 0, 0, 0, 0, …, 0]^k

Machine Learning: How to setup data

Feature Extraction

X Y

One-hot Encoding

- Each word gets an index in the vector
- All indices 0 except present word:

Feature example: which is previous word?

“Corpus”

raw data: sequences of characters

The book was interesting so I was happy .

Data

`[0, 1, 0, 0, 0, …, 0, 0, 0, 0, 0, 0, …, 0]`^{k}

`[0, 0, 1, 0, 0, …, 0, 0, 0, 0, 0, 0, …, 0]`^{k}

Machine Learning: How to setup data

Feature Extraction

X Y

Multiple One-hot encodings for one observation

(1) word before; (2) word after

"Corpus"

The book was ***interesting*** *so I was happy .*

raw data: sequences of characters

`[0, 0, 0, 0, 1, 0, …, 0]`^{k} `[0, …, 0, 1, 0, …, 0]`^{k}

Data

Machine Learning: How to setup data

Feature Extraction

X Y

Multiple One-hot encodings for one observation

(1) word before; (2) word after

*The book was **interesting** so I was happy .*

`[0, 0, 0, 0, 1, 0, …, 0]`^{k} `[0, …, 0, 1, 0, …, 0]`^{k}

=

`[0, 0, 0, 0, 1, 0, …, 0, 0, …, 0, 1, 0, …, 0]`^{2k}

Machine Learning: How to setup data

Feature Extraction

X Y

Multiple One-hot encodings for one observation

(1) word before; (2) word after; (3) percent capitals

“Corpus”

*The book was **Interesting** so I was happy .*

raw data: sequences of characters

$[0, 0, 0, 0, 1, 0, \ldots, 0]^k$ $[0, \ldots, 0, 1, 0, \ldots, 0]^k$

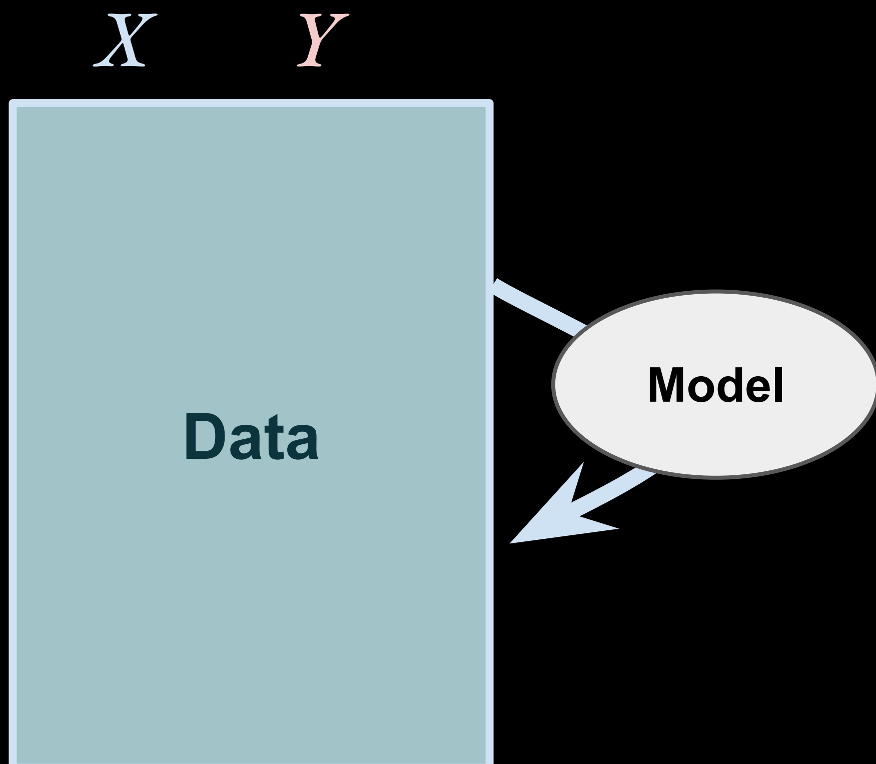
Data

=

$[0, 0, 0, 0, 1, 0, \ldots, 0, 0, \ldots, 0, 1, 0, \ldots, 0]^{2k}$

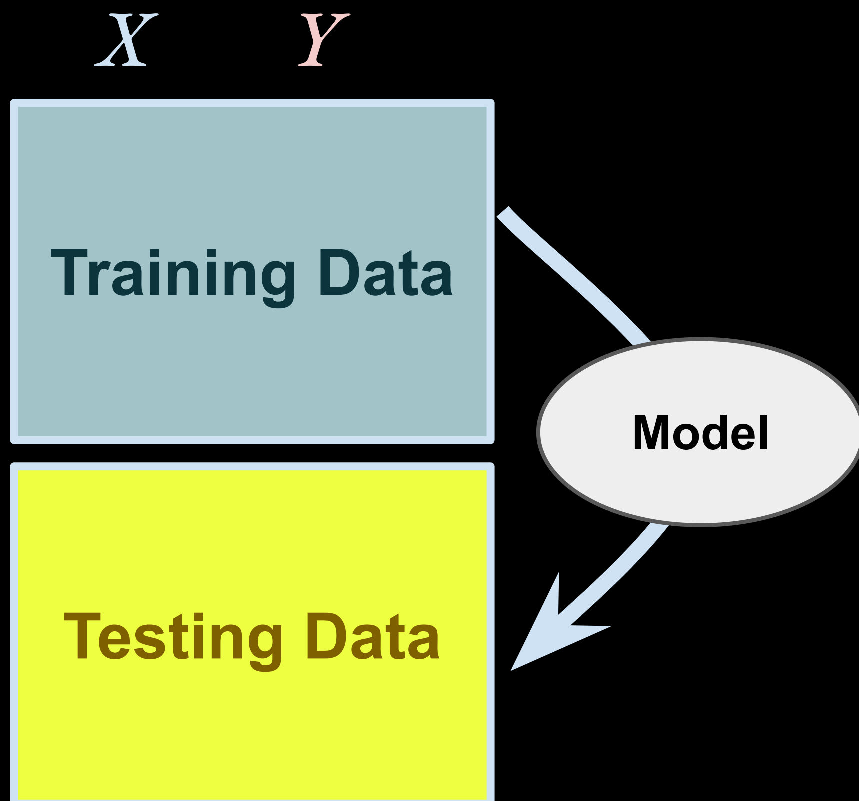
$[0, 0, 0, 0, 1, 0, \ldots, 0, 0, \ldots, 0, 1, 0, \ldots, 0, 0.09]^{2k+1}$

Machine Learning: How to setup data



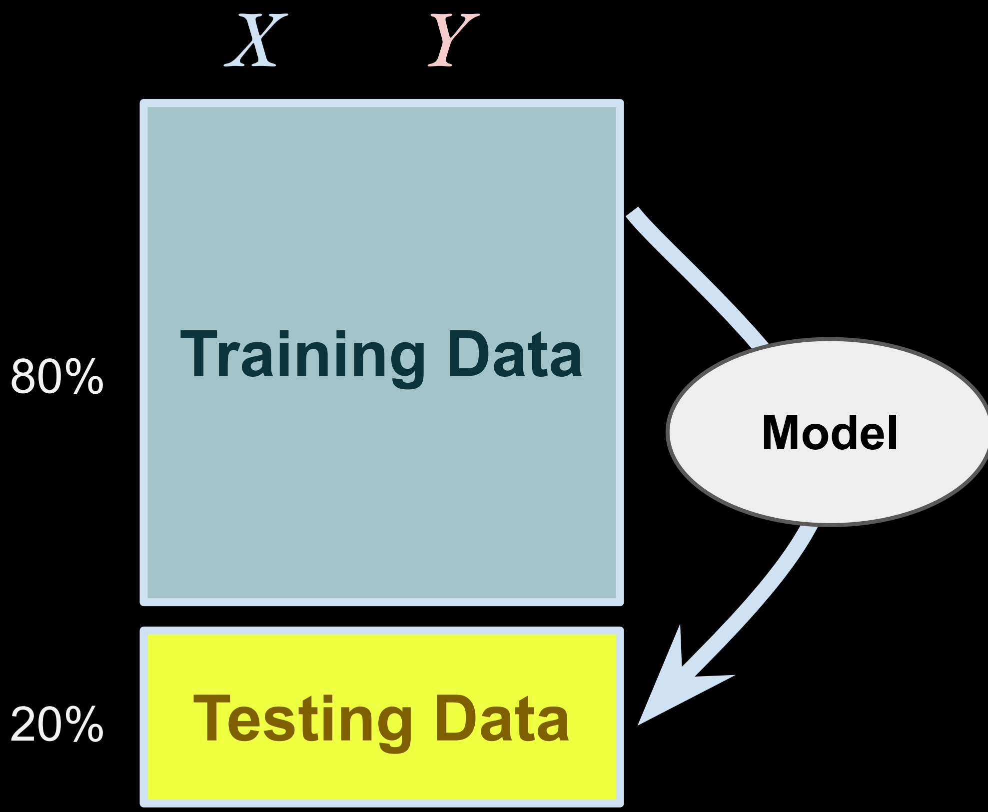
Does the
model hold up?

Machine Learning Goal: Generalize to new data



Does the model hold up?

Machine Learning Goal: Generalize to new data



Does the model hold up?

Logistic Regression - Regularization

$$X = Y$$

0.5	0	0.6	1	0	0.25	1
0	0.5	0.3	0	0	0	1
0	0	1	1	1	0.5	0
0	0	0	0	1	1	0
0.25	1	1.25	1	0.1	2	1

Logistic Regression - Regularization

$X = Y$

0.5	0	0.6	1	0	0.25	1
0	0.5	0.3	0	0	0	1
0	0	1	1	1	0.5	0
0	0	0	0	1	1	0
0.25	1	1.25	1	0.1	2	1

Logistic Regression - Regularization

X $=$ Y

x_1	x_2	...				Y
0.5	0	0.6	1	0	0.25	1
0	0.5	0.3	0	0	0	1
0	0	1	1	1	0.5	0
0	0	0	0	1	1	0
0.25	1	1.25	1	0.1	2	1

$$1.2 + -63*x_1 + 179*x_2 + 71*x_3 + 18*x_4 + -59*x_5 + 19*x_6 = logit(Y)$$

Logistic Regression - Regularization

$$X = Y$$

x_1	x_2	...				Y
0.5	0	0.6	1	0	0.25	1
0	0.5	0.3	0	0	0	1
0	0	1	1	1	0.5	0
0	0	0	0	1	1	0
0.25	1	1.25	1	0.1	2	1

$$1.2 + -63*x_1 + \boxed{179*x_2} + 71*x_3 + 18*x_4 + -59*x_5 + 19*x_6 = logit(Y)$$

Logistic Regression - Regularization

$$X = Y$$

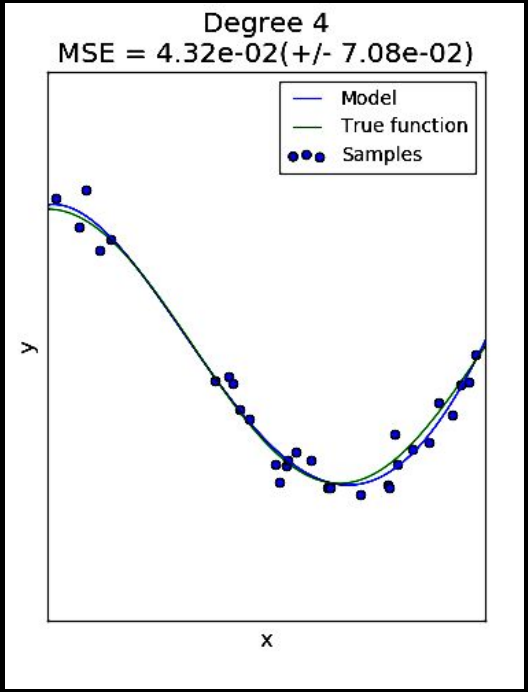
x_1	x_2	…				Y
0.5	0	0.6	1	0	0.25	1
0	0.5	[illegible]	[illegible]	[illegible]	0	1
0	0	[illegible]	[illegible]	[illegible]	0.5	0
0	0	0	0	1	1	0
0.25	1	1.25	1	0.1	2	1

“overfitting”

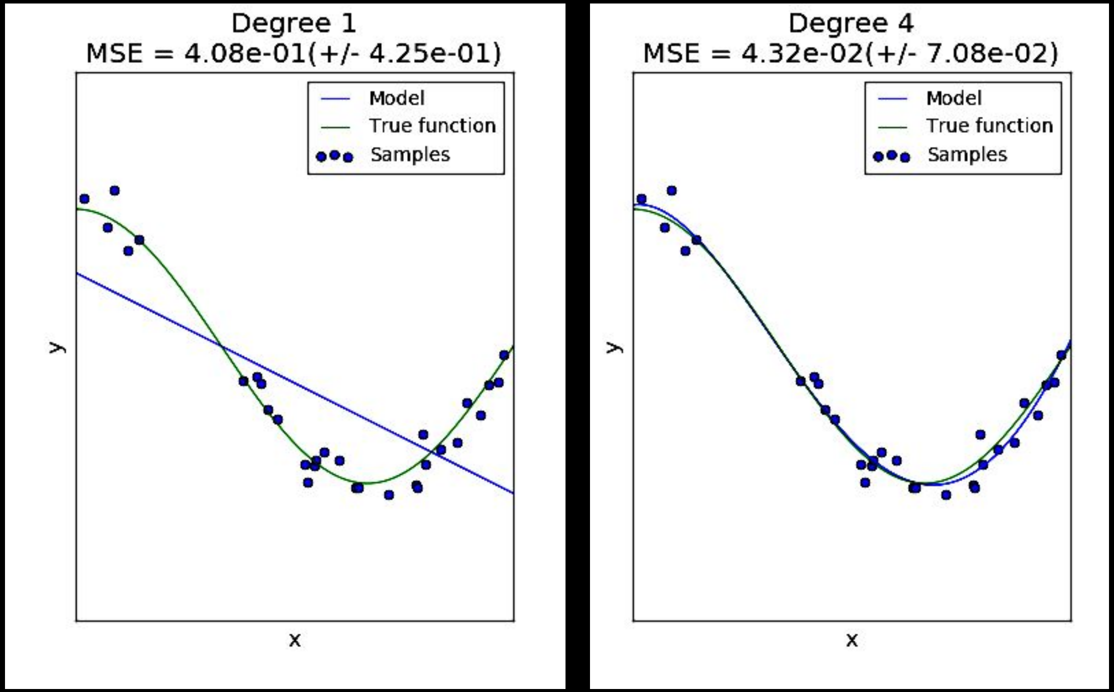
$$1.2 + -63{*}x_1 + \boxed{179{*}x_2} + 71{*}x_3 + 18{*}x_4 + -59{*}x_5 + 19{*}x_6 = logit(Y)$$

Python Example

Overfitting (1-d non-linear example)



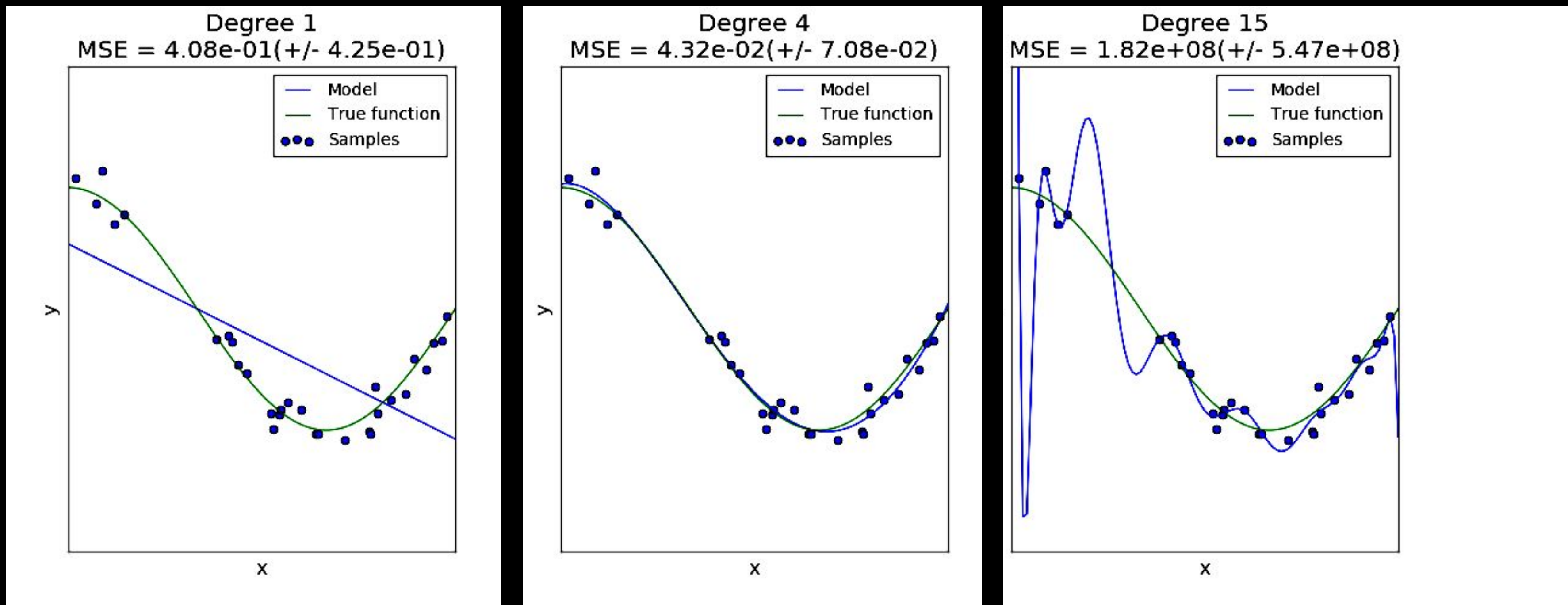
Overfitting (1-d non-linear example)



Underfit

(image credit: Scikit-learn; in practice data are rarely this clear)

Overfitting (1-d non-linear example)



Underfit

Overfit

(image credit: Scikit-learn; in practice data are rarely this clear)

Logistic Regression - Regularization

$$X = Y$$

x_1	x_2	...				Y
0.5	0	0.6	1	0	0.25	1
0	0.5	[illegible]	0	[illegible]	0	1
0	0	[illegible]	[illegible]	[illegible]	0.5	0
0	0	0	0	1	1	0
0.25	1	1.25	1	0.1	2	1

“overfitting”

$$1.2 + -63 * x_1 + \boxed{179 * x_2} + 71 * x_3 + 18 * x_4 + -59 * x_5 + 19 * x_6 = logit(Y)$$

Logistic Regression - Regularization

X

x_1	x_2
0.5	0
0	0.5
0	0
0	0
0.25	1

$= \quad Y$

1
1
0
0
1

What if only 2 predictors?

Logistic Regression - Regularization

X = Y

x_1	x_2
0.5	0
0	0.5
0	0
0	0
0.25	1

Y
1
1
0
0
1

What if only 2 predictors?

A: better fit

$0 + 2*x_1 + 2*x_2 = logit(Y)$

Logistic Regression - Regularization

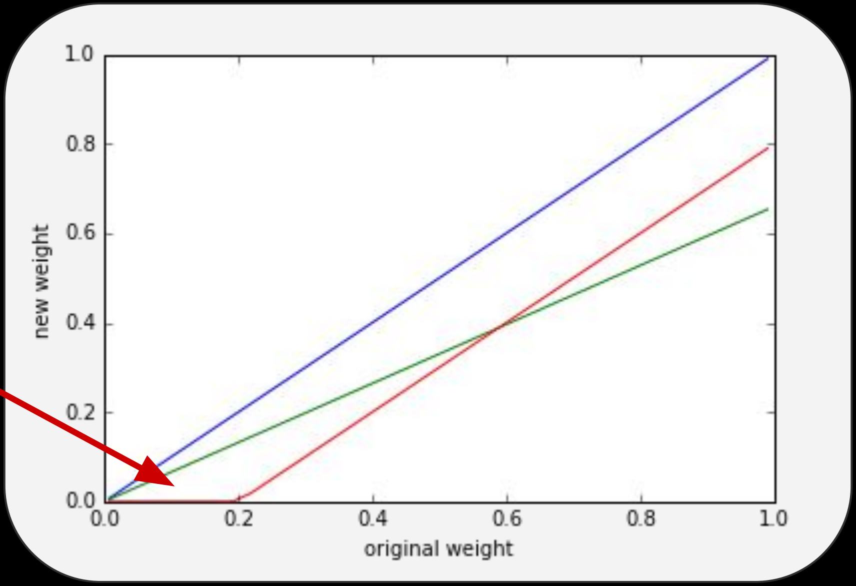
L1 Regularization - "The Lasso"

Zeros out features by adding values that keep from perfectly fitting the data.

Logistic Regression - Regularization

L1 Regularization - “The Lasso”

Zeros out features by adding values that keep from perfectly fitting the data.



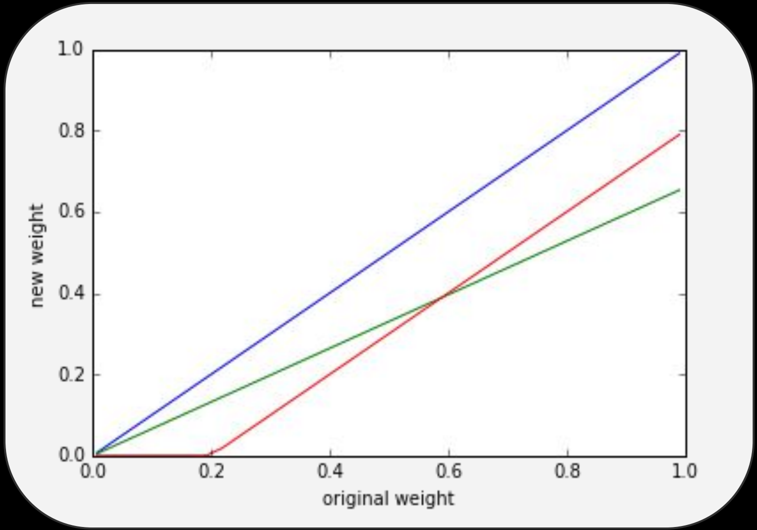
Logistic Regression - Regularization

L1 Regularization - “The Lasso”

Zeros out features by adding values that keep from perfectly fitting the data.

$$L(\beta_0, \beta_1, ..., \beta_k | X, Y) = \prod_{i=1}^{n} p(x_i)^{y_i}(1 - p(x_i))^{1-y_i}$$

set betas that maximize L



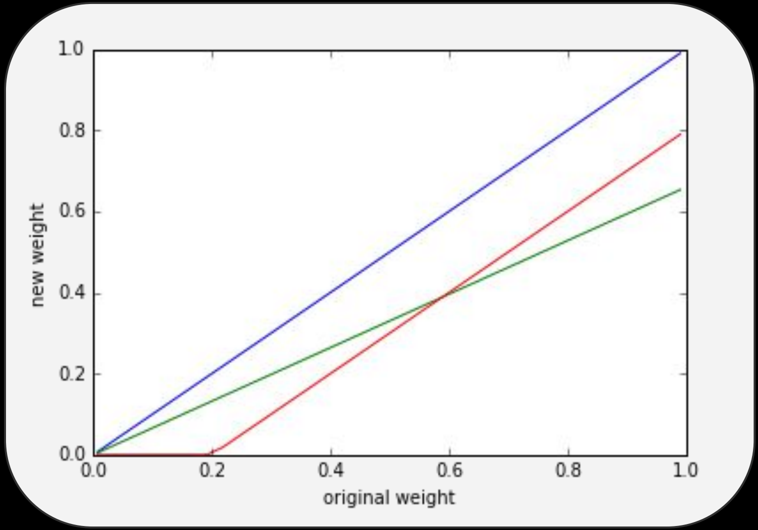
Logistic Regression - Regularization

L1 Regularization - "The Lasso"

Zeros out features by adding values that keep from perfectly fitting the data.

$$L(\beta_0, \beta_1, ..., \beta_k | X, Y) = \prod_{i=1}^{n} p(x_i)^{y_i}(1 - p(x_i))^{1-y_i} - \frac{1}{C}\sum_{j=1}^{m} |\beta_j|$$

set betas that maximize *penalized* L



Logistic Regression - Regularization

Sometimes written as:

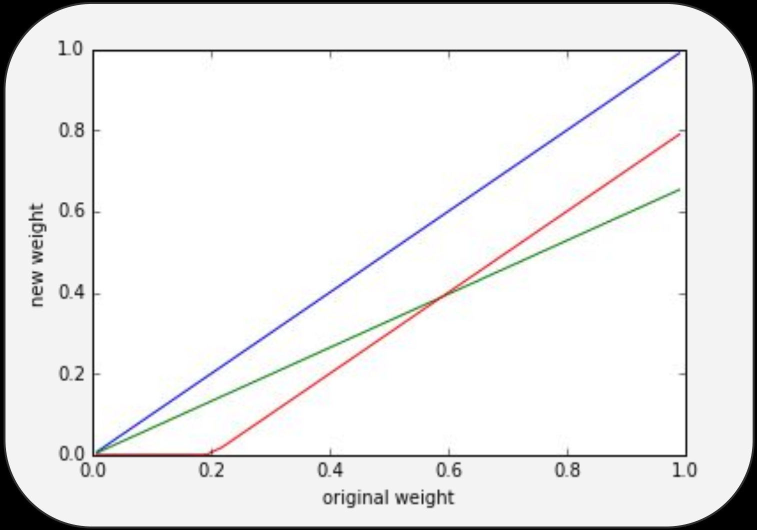
$$||\beta||_1$$

L1 Regularization - "The Lasso"

Zeros out features by adding values that keep from perfectly fitting the data.

$$L(\beta_0, \beta_1, \ldots, \beta_k|X,Y) = \prod_{i=1}^{n} p(x_i)^{y_i}(1-p(x_i))^{1-y_i} - \frac{1}{C}\sum_{j=1}^{m}|\beta_j|$$

set betas that maximize *penalized* L



Logistic Regression - Regularization

Sometimes written as:

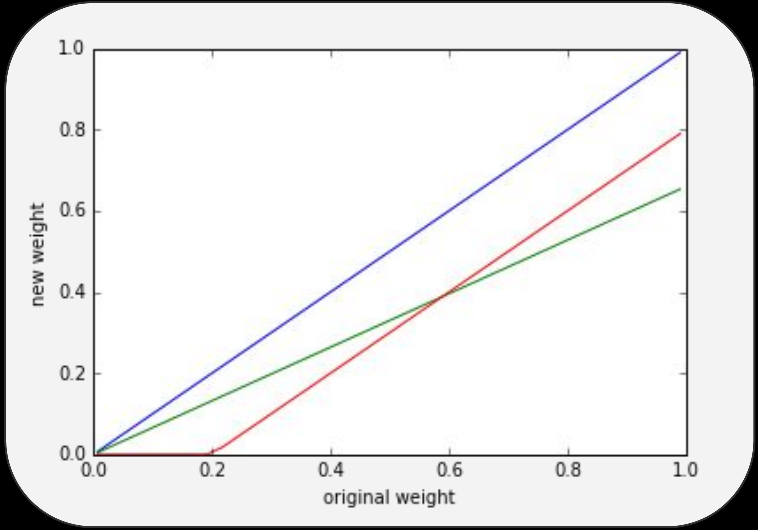
$$||\beta_j||_2^2$$

L2 Regularization - "Ridge"

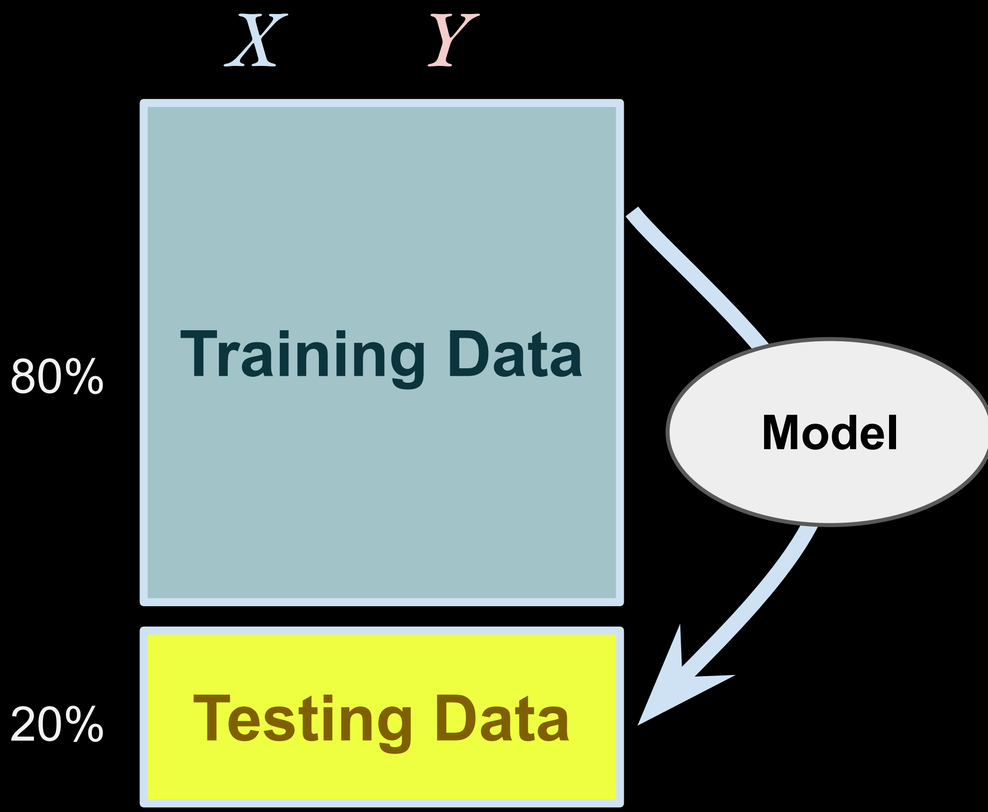
Shrinks features by adding values that keep from perfectly fitting the data.

$$L(\beta_0, \beta_1, \ldots, \beta_k | X, Y) = \prod_{i=1}^{n} p(x_i)^{y_i}(1 - p(x_i))^{1-y_i} - \underbrace{\frac{1}{C}\sum_{j=1}^{m} \beta_j^2}$$

set betas that maximize *penalized* L

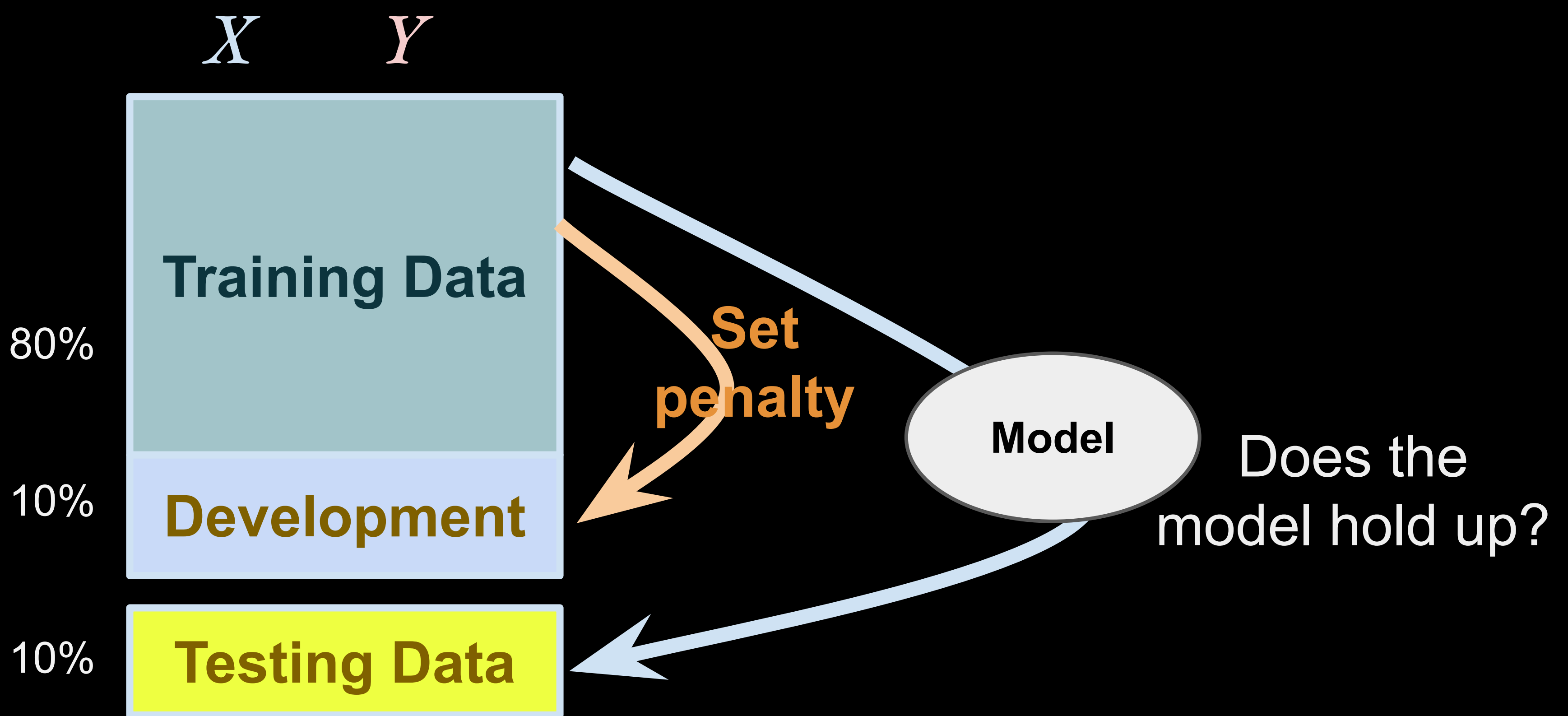


Machine Learning Goal: Generalize to new data



Does the model hold up?

Machine Learning Goal: Generalize to new data



Logistic Regression - Review

- Classification: $P(Y \mid X)$
- Learn logistic curve based on example data
  - training + development + testing data
- Set betas based on maximizing the likelihood
  - "shifts" and "twists" the logistic curve
- Multivariate features: One-hot encodings
- Separation represented by hyperplane
- Overfitting
- Regularization

Example

See notebook on website.

```
In [44]: %matplotlib inline

         #above allows plots to discplay on the screen.

         #python includes
         import sys

         #standard probability includes:
         import numpy as np #matrices and data structures
         import scipy.stats as ss #standard statistical operations
         import pandas as pd #keeps data organized, works well with data
         import matplotlib
         import matplotlib.pyplot as plt #plot visualization
```

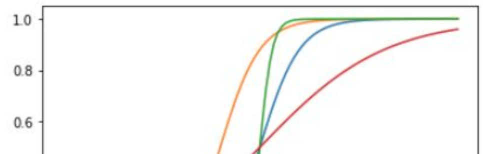
```
In [53]: #Let's just look at what happens to the logit function as we change the beta coefficients

         def logistic_function(x):
             return np.exp(x) / (1+np.exp(x))

         def logistic_function_with_betas(x, beta0=0, beta1=1):
             #now using linear function: beta0 + beta1*x for the exponent:
             return np.exp(beta0 + beta1*x) / (1+np.exp(beta0 + beta1*x))

         xpoints = np.linspace(-10, 10, 100)
         plt.plot(xpoints, [logistic_function(x) for x in xpoints])
         plt.plot(xpoints, [logistic_function_with_betas(x, 2, 1) for x in xpoints]) #shifts the intercept with zero
         plt.plot(xpoints, [logistic_function_with_betas(x, 0, 3.145914159653) for x in xpoints])#twists the line verically
         plt.plot(xpoints, [logistic_function_with_betas(x, 0, 1/3.145914159653) for x in xpoints]) #twists it horizontally
```

```
Out[53]: [<matplotlib.lines.Line2D at 0x2691f435f60>]
```



Extra Material

One approach to finding the parameters which maximize the likelihood function...

"best fit" : whatever maximizes the likelihood function:

$$L(\beta_0, \beta_1, \ldots, \beta_k | X, Y) = \prod_{i=1}^{n} p(x_i)^{y_i}(1 - p(x_i))^{1-y_i}$$

$$p_i \equiv P(Y_i = 1 | X_i = x) = \frac{e^{\boxed{\beta_0}+\boxed{\beta_1}x_i}}{1 + e^{\boxed{\beta_0}+\boxed{\beta_1}x_i}}$$

To estimate β, one can use *reweighted least squares:*

(Wasserman, 2005; Li, 2010)

set $\hat{\beta}_0 = \ldots = \hat{\beta}_m = 0$ (remember to include an intercept)

1. Calculate p_i and let W be a diagonal matrix where element$(i, i) = p_i(1 - p_i)$.
2. Set $z_i = logit(p_i) + \frac{Y_i - p_i}{p_i(1 - p_i)} = X\hat{\beta} + \frac{Y_i - p_i}{p_i(1 - p_i)}$
3. Set $\hat{\beta} = (X^T W X)^{-1} X^T W z$ //weighted lin. reg. of Z on Y.
4. Repeat from 1 until $\hat{\beta}$ converges.

"best fit" : whatever maximizes the likelihood function:

$$L(\beta_0, \beta_1, \ldots, \beta_k | X, Y) = \prod_{i=1}^{n} p(x_i)^{y_i}(1 - p(x_i))^{1-y_i}$$

This is just one way of finding the betas that maximize the likelihood function. In practice, we will use existing libraries that are fast and support additional useful steps like **regularization**..

To estimate β , one can use *reweighted least squares:*

(Wasserman, 2005; Li, 2010)

set $\hat{\beta}_0 = \ldots = \hat{\beta}_m = 0$ (remember to include an intercept)

1. Calculate p_i and let W be a diagonal matrix where element$(i, i) = p_i(1 - p_i)$.
2. Set $z_i = logit(p_i) + \dfrac{Y_i - p_i}{p_i(1-p_i)} = X\hat{\beta} + \dfrac{Y_i - p_i}{p_i(1-p_i)}$
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